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THEORETICAL ANALYSIS OF FUZZY LOGIC AND Q. E. METHOD IN ECONOMICS

This paper analyzes the key elements of fuzzy logic and showes that through rational, behavioral economics and neo-classical economics it is possible to develop models using the Q. E. methodology. Therefore, it is plausible to apply contemporaneous Q. E. methodology in combination with the rationality and the behavioral approach. The fuzzy logic and the generator is the source of this mechanism for the production of the appropriate models.

Анализируются ключевые элементы нечеткой логики. Показано, что с помощью рациональной, поведенческой и неоклассической политэкономии можно разработать модели с использованием методологии количественного определения. Следовательно, вполне вероятно, что методология будет результативна в сочетании с рационально-поведенческим подходом с использованием количественного определения. Нечеткая логика и генеративность являются источниками этого механизма для производства соответствующих моделей.

Keywords: fuzzy logic, quantification method, rational, behavioral, Q. E. method.

Ключевые слова: нечеткая логика, метод квантификации, рационально-поведенческий метод, метод количественного определения.

1. Introduction. Basic principles of fuzzy logic

The fuzzy logic represents the relationship between precision and uncertainty. As the uncertainty in any issue is high, then less precise we can be in our conception. A binary logic admits only the opposites of true and false, a logic which does not admit digress of truth and there are no variations in magnitudes, but only two possible results. As more complex is a system, then more imprecise or inexact is the information that we have to the system. Aristotle mentioned that "It is the mark of an instructed mind to rest satisfied with that degree of precision which the nature of the subject admits, and not to seek exactness where only an approximation of the truth is possible" [1; 15]. So, Aristotelian logic does not admit imprecision in truth. However, Aristotle's quote is relevant to the approach that admits uncertainty [16]. The theme is the balance between the precision with the uncertainty as a concept [5]. The case of imprecision comes from physical processes upon on imprecise human reasoning. Requiring precision in engineering models and economics means high cost and long lead times in production and development. So, considering the use of fuzzy logic then ponder the need for exploiting the tolerance for imprecision. According to the traditional view of science, uncertainty represents an undesirable situation and must be excluded at any cost.

Max Black referred to vagueness, where the possible states are not clear. According to his essay in 1937 known as "Vagueness: An exercise in logical analysis" presented some remarks by Plato about Uncertainty in geometry. Russell in 1923 pointed out that "all traditional logic habitually assumes that precise symbols are being employed". So, there are some proposals. In 1965 Lofti A Zadeh in his paper "Fuzzy Sets" he introduced for the first time the concept of fuzzy logic. The main idea of fuzzy sets and fuzzy logic were not accepted in many cases because some of the underlying mathematics had not yet been explored. Based on the lack of mathematical development in the Western world the usage of fuzzy logic was not so wide. But, in Japan was fully accepted no matter for the initial condition that was stated. In Japan fuzzy logic implemented in many technological products, and the reason was only that fuzzy logic fully worked, regardless if mathematic proofs existed or not. In the decade of 1980s Japanese products used fuzzy logic algorithms and facts, and in the U.S. this happened at the end of this decade. To show the difficulties that fuzzy logic faced we must refer to the fact that a colleague of Lofti A Zadeh, named William Kahan had a lot of objections for this theory because he didn't accept the imprecise thinking that fuzzy logic consists. One similar approach comes from Aristotle and his predecessors who devised their theories of logic and mathematics, and according to them has suggested the Law of the Excluded Middle. The law of non-contradiction tells us that a statement cannot be both true and false at the same time. The law of excluded middle can be expressed by a formula that means that a statement is either true or false. Based on the concept of the fuzzy logic of Lofti A Zadeh, we use the more recent improvements of this theory, of fuzzy logic with respect to the ambient of Artificial Neural Networks and Machine Learning. A new view of fuzzy logic could lead to solutions that assumptions of neoclassical points of view and behavioral economics face.

2. Methodology — The Q. E. method

The axiomatics is used extensively to the field of economics. The axiomatics stands on the assumption that we don't know the result of one hypothesis, and therefore the confirmation that result is sustainable with the hypothesis is the key concept of this approach [6]. Hence, the hypothesis is used for the determination of the results. The hypothesis of an economic theory is the basis for the further study of each economic model that is under examination. Therefore, the axiomatics is trying to answer to the background of an economic analysis and to confirm that the initial hypothesis of the model is satisfied. If the hypothesis is satisfied then the model is consistent with the principles of the model that is under examination. Thence, we have two cases about the axiomatics:

• To the first case of the axiomatics the results comply with the hypothesis, and then the model is satisfied. To that fact, the examination could stop as there is the reliability of the theory is that which scientist perceived.

• The second one case is about the incident that the axiomatics is not satisfied, because the initial hypothesis of the model is not satisfied. Thence, the scientist should proceed to adjustments of the model, until to fit to be the hypothesis in respect to the results [3].

Therefore, in this case, we conclude that the economic model is not sufficient. Then, the main concept of axiomatics stands on the correspondence of the initial hypothesis to the mathematical and economic result of the scrutiny. Inasmuch as is plausible to the scientist to clarify the theory about the chosen model.

The theme of the Q.E. theory is based on a methodology which stands on the determination of mathematic equations subject to conditions which also considered. One more important thing is the determination of the upper and the lower limit of the values of the independent variables. Forasmuch as, the dependent variable represents the behavior of the selected model, pending on a generator which produces random values to all the independent variables to configure the interaction between them and their behavior under different conditions. At least the basic study includes two facets which are:

• The analysis of the behavior of the model which stands on the scrutiny of the structural characteristics of each model accordingly allowing with that way the extraction of general conclusions about the model which is under examination. There is plausible the analysis of data into a two and to three-dimensional analysis pending on the case.

• The frequency analysis behavior scrutinizes the behavior of the dependent variables, but from the view of the number of appearances of a variable than another, estimating basically the impact that one independent variable has with one or more others independent variables [4].

The dependent variables are these which are modified for the generator. Thereupon, the generator produces values for the dependent variables. The extracted values of the generator allow the creation of magnitudes, which are the base for comparisons, and for the analysis of mathematical equations. With that way is plausible to quantify quality data and theoretical terms. Moreover, according to this methodology, the created magnitudes allow proceeding furthermore to econometrical analyses. In general is a methodology for quantification of quality data. Thus, using the Quantification of Everything (Q.E.) methodology is plausible to clarify the behavior of any model and to determine its standalone behavior, or its comparative behavior, between different models [7]. Hence, the Q.E. methodology follows as index four basic steps. These steps are described below:

• The first step is about the element of the hypothesis. At this point is determined the aim and the scope of the analysis of each study. Thus, the mathematical determination is the main point of this step.

• The second step is about the generator, which produces the values for the independent variable. Should, be notified that in this procedure is taken under consideration the upper and the lower limit, which used for the production of values through the randomization process. Thence, we are able to proceed to the formation of the quality data into a quantified form. Thence, through irritations is plausible to clarify the appropriate mathematical equation.

• The third step is about the conclusions. To this step, the scientist is able to extract the adequate exegesis about the form of the model. According to this formation, adequacy is pending the further recodification of the examination model.
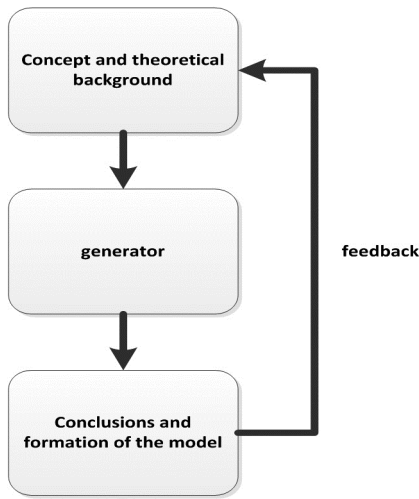
Fig. 1. Method of quantification
of quality data

• The fourth step is about the feedback and is the connector between the conclusions with the aims and scopes of the initial hypothesis. Thence, the iterations are the source of the adequacy of the model.

Therefore, we have the scheme (fig. 1). This scheme has a full analysis of the paper «Methods of Controlled Transactions and the Behavior of Companies According to the Public and Tax Policy» [2].

Thence, using that methodology, we are able to proceed to the study of the behavior of the model. The four steps of the Q. E. methodology are fully described by the prior figure, showing that the feedback is the source of the readjustments of the model.

3. Fuzzy logic concepts

To the fuzzy logic, concepts offered some views on fuzzy sets. There is an intention to share information about remarkable researchers and humans, highlighting their long-lasting research accomplishments and stressing with that way a visible impact on the fuzzy set community. These researchers have shown the new and promising research potentials initiated by fuzzy regression and they identified future developments of these models emerging within the Granual Computing. Moreover, giving a variety of granular fuzzy models, including and the case of higher-order and higher-type granular constructs enhanced this approach [14]. To the case of economics, a similar usage of fuzzy logic could be found an application through the Q. E. method, which should be considered as appropriate for this establishment of fuzzy logic to the social sciences.

To this case proposed that the fuzzy joint points (FJP) method is one of the most successful approaches to destiny-based clustering. Except for the basic FJP method, there are other methods like the Noise-Robust FJP (NRFJP), and fuzzy Neighborhood DBSCAN (FN-DBSCAN). But, these methods were slow according to the FJP algorithm, inducing the problem of large databases. The Modified FJP (MFJP) addresses that issue and achieves an improvement in speed [11]. But, the problem with this method was at the point of applicability. So, to face that issue had integrated several methods with FJP to establish an optimal-time algorithm [13]. Like to the case of FJP, a similar methodology could be implemented by an adequate mathematical background, which could allow a kind of algorithm that would comply with economics; with main aim the clarification of an optimization, which would be able to find the best approach to face the deviations between neoclassical economics and behavioral economics.

The concept to the theory of fuzzy transform (F-transform) is the fuzzy partition, which is a generalization of the classical concept of partition. The original definition assumes that every two normal fuzzy subsets in a partition overlap, in such a way that the sum of the membership degrees at each point is equal to one. Therefore it is plausible to relax the assumption of normality for fuzzy sets. The result is a denser fuzzy partition that improves the approximation properties and the smoothness of the inverted F-transform. Using that approach properly we would be able to proceed to fuzzy partitions which would be referred to us, as general partitions. A kind of partitions is plausible to find tools that would enhance the general partitions of fuzzy logic adjusted to the case of economics. Thence, would have better solutions to economic and to financial issues [10].

Moreover, in this field of study had proposed a new method for building adaptive neuro-fuzzy inference systems (ANFIS) via databases. To improve the performance of conventional ANFIS is to handle noisy data [17]. The handling of these noisy data is focused on the improvement of a given dataset. Therefore, according to that thesis had proposed a weighted clustering process subject to input-output data space. During the clustering process, the cluster with the smallest potential distance has priority when obtained the surveyed sample. This formulation based on the fact that this procedure stands on a combination of the Euclidean distance and of the size of clusters. Then, the impact of undetermined coefficients and of factors of economic theories that have not included magnitudes to their estimations, affect the results, but the same time influences the resolution of the problem. Hence, the approach of ANFIS could be adjusted to those problems.

The mainstream idea is to emphasize the use of fuzziness in data analysis, to be able to capture information that has been traditionally disregarded with a cost to the precision of the conclusions. According to that epistemic approach, the main theme is the fuzziness of data and not so much the data analysis process which used in various stages. To this work proposed a general and intuitive method to discard outliers. The absence of some important factors must be taken into account to deal with the lack of the precision of the economic target and based on that the fuzziness could achieve better results from the exactness of the data. To this thesis is showed the examination of the fuzzy querying as a human consistent and in addition showed a friendly way of retrieving information, due to real human intentions and preferences; which expressed in natural language represented with the usage of fuzzy logic and possibility theory [8]. To bipolar queries can be accommodated the user's intentions and preferences involving some sort of a required, desired, mandatory and optional elements. According to the bipolar theme, it is plausible to establish the optimized solutions for each case that there exist preferences of the consumers that have an unusual attitude. Moreover, according to the appropriate formula, it is plausible to estimate from the different choices of the consumers, a system that would be familiar with the case of fuzzy examination of constant choices.

This work focuses on the relationship between fuzzy sets and formal logics. Lluis Godo, Siegfried Gottwald discuss the influences of fuzzy set theory about the development of particular formal logics, emphasizing on the historical side of these structures [9]. So, would be useful to be produced

a set that is associated with economic data which comply with fuzzy logic adjustments. The Q. E. methodology offers that view, as combines economic theories that come from the neoclassical view and from experimental economic data of behavioral economics.

The main concept of Frank Klawonn, Rudolf Kruse, and Roland Winkler based on the fact that the initial idea extends the classical k-means clustering technique to an algorithm that uses membership degrees, instead of data objects to clusters that led to the invention of a large variety of new fuzzy clustering algorithms [12]. With the usage of crisp assignments of data objects to clusters, these algorithms would get stuck most of the times in a local minimum of their underlying objective functions, leading to undesired clustering results. The use of membership degrees can avoid this local minimum, but causes other problems to that clusters of varying density. The extension of adequate fuzzy algorithms to more complicated forms could permit a more sophisticated approach of solutions of fuzzy logic approaches, from the view of economic uncertainty or limited data.

Enrich Trillas presented a new way to look at Commonsense Reasoning. Since imprecision is pervasive in Commonsense Reasoning, the development of "what is presented is done" with fuzzy sets, endowed a very loose structure. According to Enrich Trillas the study of Commonsense Reasoning is one approach that researchers cannot underestimate, thinking that is not just a logical or mathematical subject, but should consider that it is something much more. The limit data cause imprecision, but from that point of view of this theory deduced the common sense of logic, which interprets the reason that fuzzy logic issues for sets help to cover these themes of limited data, solving the limitation of accuracy to economic theories [18]. Considering the experiments of behavioral economics and the disturbances which caused by the rational economics of neoclassical approaches, we are able with quantification methods to fill the gap which arises between rational and behavioral economics.

4. The application of fuzzy logic to the Q. E. method

Thence, to this thesis are described the main developments, the results, and methods which achieved in the last years, with references on different Fuzzy Linear Programming problems (with analysis on topics in the Fuzzy Sets and Systems). So, here it is plausible to underline the influence of fuzzy logic in algorithms, meaning, in general, the formulas that could be affected with the application of fuzzy logic the economic topics and the financial mechanisms [19].

The concept of this work is characterized by labels taken from a fuzzy rating scale and the special class of linguistic expressions. Vilem Novak demonstrated with his work the power of fuzzy theory and showed that it is capable to provide tools for advanced modeling of linguistic semantics. So, this approach in economics could be used to more general implementations, like to the case of the consumer, connecting their issues with semantics and behavioral attitudes [10].

Therefore, in this approach outlined a research in fuzzy machine learning which based on formal concepts and mathematical tools from fuzzy sets and fuzzy logic (in the field of machine learning). Thence are described the

previous contributions of fuzzy machine learning and are proposed some solutions for future work. Fuzzy machine learning could be transferred in economics, to type sets that are tackled by factors and variables which pending to other economic magnitudes, deducing with that way a learning system from a time sequence. Hence, we would be able to conceive the behavior of them through time or to more abstract cases without time. We obtain with that way, a formula for machine learning in the field of economics.

The machine learning in economics takes a particular form through the Q. E. method. Simultaneously the fuzzy logic enters the Q. E. method using the generator. To the next scheme we obtain the structure of our prior analysis (fig. 2).
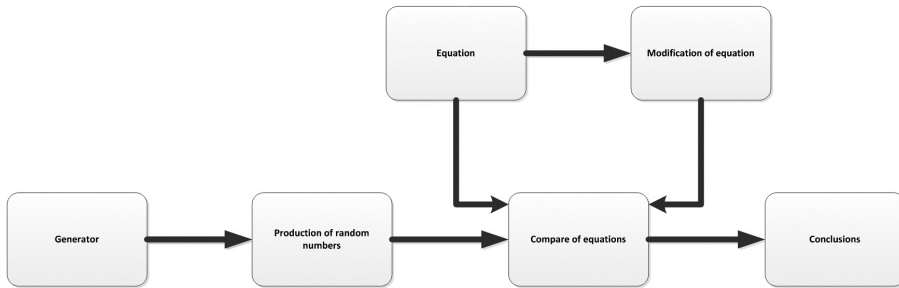
Fig. 2. Mechanism of the generator

Thence, from the prior scheme, we determine the key elements of the application of the fuzzy logic to the mechanism of the generator of the Q. E. method. Hence, we have that:

• The generator is based on the randomizing generator, showing that fuzzy logic could be used to have specific results. The generator mechanism gives the random numbers for the clarification of the appropriate equation.

• The modification of the model happens through the changes to the initial equation. The independent variables are omitted or added more variables for the comparability analysis through the initial equation and the modified equation.

• The comparison of equations and the generated numbers allow the extraction of the appropriate conclusions about the behavior of the model, which is under examination.

We have that the generator uses the fuzzy logic to determine the behavior of the model, which is under study. Moreover, according to figure 1, we seemed that there is the step of the feedback. Then, through the feedback is plausible to determine if the conclusions of the generator comply with the initial hypothesis of the model. This procedure of the feedback could be considered as a machine learning procedure of a quantified procedure, as the system learns from the adjustments and the modifications to clarify if the model is the adequate one. Thence, we perceive that the repetitions that happen through the feedback clarify the appropriate model. Thence, the condition of Q. E. method is represented by the equation:

$$m_i = m_{i-1} \pm x.$$

Where, $i = 1, \ldots, n$.

To the prior equation, *m* is about the equation which is subject to modification. In addition, sa ymbol of *i* represents the iterations which happen to cover purposes about the appropriate adjustment of the model. Moreover, the *x* is about the independent variables which added or subtracted to the model, serving the modification of the model.

An application of this concept is shown to the work "Methods of Controlled Transactions and the Behavior of Companies According to the Public and Tax Policy" where the application of fuzzy logic, of feedback and of generator gave the instance of behavior analysis and of frequency analysis.
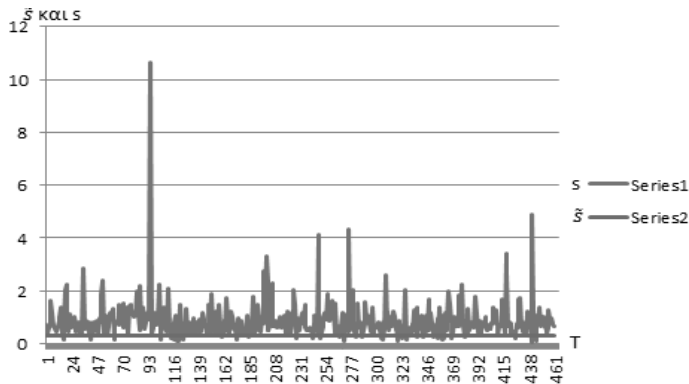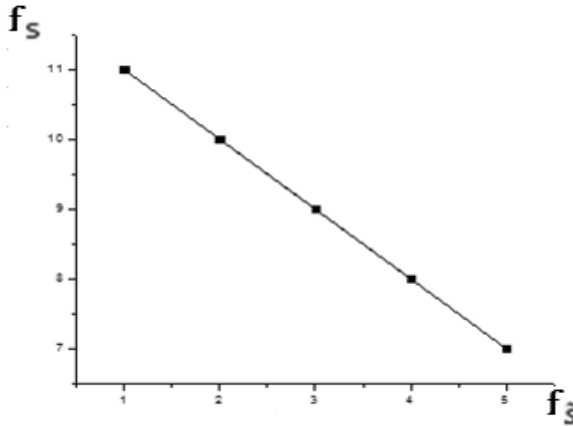
a

b

Fig. 3: *a* — behavior analysis;
b — frequency analysis (based on the results of [2])

Then, to this application had extracted conclusions from the behavioral analysis and from frequency analysis, which both of the pending to the Q. E. approach. Had extracted conclusions about:

 • The behavior analysis of the model which offers through a comparability analysis, conclusions about the model which is under examination.

• The frequency analysis behavior, which offers conclusions about the impact that one independent variable, has to one or more others independent variables [2].

Then we obtain that the fuzzy logic and the elements of Q.E. method clarify the adequacy of the model.

5. Conclusions

So, from the application of fuzzy logic we obtain that is plausible to introduce theoretical aspects that are compatible with models of rationality, to extend them with behavioral economics and quantified by the application of the Q.E. method; to be able to extract the appropriate results. The source of this anticipation is the perception of the uncertainty that behavioral economics and fuzzy logic offer in combination with the certainty of rational models. Thence, the multifunction of rational, behavioral and quantified techniques offers the examination and the development of appropriate models.

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