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1 + 1-DIMENSIONAL YANG — MILLS EQUATIONS AND MASS VIA QUASICLASSICAL CORRECTION TO ACTION

Two-dimensional Yang — Mills models in a pseudo-euclidean space are considered from a point of view of a class of nonlinear Klein — Gordon — Fock equations. It is shown that the Nahm reduction does not work, another, novel choice is proposed and investigated. A quasiclassical quantization of the models is based on Feynmann — Maslov path integral construction and its zeta function representation in terms of a Green function diagonal for an auxiliary heat equation with an elliptic potential. The natural renormalization use a freedom in vacuum state choice as well as the choice of the norm of an evolution operator eigenvectors. A nonzero mass appears as the quasiclassical correction, that is expressed via hyperelliptic integral.

Двумерные модели Янга — Миллса в псевдоевклидовом пространстве рассматриваются с точки зрения одного класса нелинейных уравнений Клейна — Гордона — Фока. Показано, что уменьшение Нама не работает, предложен и исследован другой, новый выбор. Квазиклассическое квантование моделей основано на построении интеграла по траекториям Фейнмана — Маслова и представлении его дзета-функции в виде диагональной функции Грина для уравнения вспомогательной теплоты с эллиптическим потенциалом. При естественной перенормировке используется свобода выбора состояния вакуума, а также выбор нормы собственных векторов оператора эволюции. Ненулевая масса появляется как квазиклассическая поправка, которая выражается через гиперэллиптический интеграл.

Keywords: Yang — Mills equations, nonlinear plane wave, Green function diagonal.

Ключевые слова: уравнения Янга — Миллса, нелинейная плоская волна, диагональ функции Грина.

1. Introduction. On Nahm models

Underlying ideas for this investigation, related to the classical Yang — Mills (YM) theory reductions, were taken from works of Baseyan [3], Corrigan [6] and Nahm [8].

This paper is a direct development of author's results [10] in which one-dimensional model, immersed in SU(2) YM theory, was studied in the context of Nahm model. The author's main result [10] is a demonstration of existence and evaluation of nonzero quantum correction to action against classical zero energy (representing mass) as a consequence of the proposed model. The one-dimensional Yang — Mills — Nahm models were considered from algebrogeometric points of view. A quasiclassical quantization of the models is based on Maslov version of path integral construction and its zeta function representation in terms of a Green function diagonal for an auxiliary heat equation with an elliptic potential. The Green function diagonal and, hence, the generalized zeta function and its derivative are expressed via solutions of Drach equation [15] and, alternatively, by means of Its — Matveev [19] formalism in terms of Riemann theta-function. The approach is based on Baker-Akhiezer functions for Kadomtsev — Petviashvili equation [12]. The quantum corrections to action of the model are evaluated. The fields from the class of elliptic functions are properly studied. For such model, which field is represented via elliptic (lemniscate) integral by construction, YM field mass is defined as the quantum correction, in the quasiclassical approximation it is evaluated via hyperelliptic integral.

The model, via the Atiyah — Drinfeld — Hitchin — Manin — Nahm (ADHMN) construction of static monopole solutions, is related to Yang — Mills — Higgs theories in four dimensions in the Bogomolnyi — Prasad — Sommerfield limit. The ADHMN construction ⇔ — equivalence between self-dual equations, one — unidimensional, the other in three dimensions (reduced Euclidean four dimensional theory by deleting dependence on a single variable), see [16].

The weak point of description starting from the 1 + 0 Nahm model is namely the one-dimensionality of the reduction that provoke ambiguity of the interpretation of such correction as the mass.

Let us list basic elements of this paper construction.

1. Yang — Mills equations in PseudoEuclidean dimensions. The equation for YM field T_μ from semisimple compact gauge group in covariant form reads as

$$\nabla^{\mu} T_{\mu\nu} = 0, \tag{1}$$

whence $\mu, \nu = 0,1,2,3$, the time variable is $x_0 = ct, c = 1;\ x_k$ — space variables. For the gauge fields $T_\mu = T_\mu^+$, where

$$T_{\mu\nu} = \partial_\mu T_\nu - \partial_\nu T_\mu - [T_\mu, T_\nu], \quad \partial_\nu \Phi = \partial_\mu - [T_\mu, \Phi], \tag{2}$$

one has the covariant equation

$$\begin{gathered}\Box T_\nu - \partial_\nu \partial_\mu T_\mu + [T_\mu, \partial_\nu T_\mu - \partial_\mu T_\nu] + [T_\mu, [T_\mu, T_\nu]] - \\ -\partial_\mu [T_\mu, T_\nu] = 0,\end{gathered} \tag{3}$$

as written, e. g., in Faddeev — Slavnov book [2].

The reduction via independence on x_k, k = 1, 2, 3; setting $x_0 = t$, choosing the Hamilton gauge $T_0 = 0$, gives

$$\frac{d^2 T_k}{dt^2} = [T_j [T_j, T_k]], \quad [T_k, \frac{dT_k}{dt}] = 0. \tag{4}$$

The self-dual equations [6],

$$\frac{dT_i}{dt} = \pm\varepsilon_{ijk} T_j T_k, \tag{5}$$

imply Eqs. (4).

For illustration we would use 2 x 2 matrix gauge group (isospin group SU(2)) and the basis of Pauli matrices σ_i , expanding $T_\mu = A_\mu^k \sigma_k$. Equalizing terms by σ_k and evaluating sums one goes to the vector form

$$\begin{gathered}\Box A_\nu^k - \partial_\nu \partial_\mu A_\mu^k + A_\mu^j \varepsilon_{jpk} (i\partial_\nu A_\mu^p - i\partial_\mu A_\nu^p) - \\ -iA_\mu^j A_\mu^j A_\nu^k + iA_\mu^j A_\mu^k A_\nu^j - i\varepsilon_{jpk} A_\nu^p \partial_\mu A_\mu^j - i\varepsilon_{jpk} A_\mu^j \partial_\mu A_\nu^p.\end{gathered} \tag{6}$$

2. YM equations: vector form, Lorentz gauge. Rescaling the vector potential we return the self-action charge parameter ε to rewrite the YM equation keeping the same notations

$$\Box \vec{A}_k + 2\varepsilon \vec{A}^\mu \times (2\partial_\mu \vec{A}_k - \partial_k \vec{A}_\mu - 2\varepsilon \vec{A}_k \times \vec{A}_\mu) = 0, \tag{7}$$

$k = 1,2,3,$ where, $\vec{A}_0$ is expressed from the Lorentz gauge

$$\vec{A}_0 = \partial_0^{-1} \partial_k \vec{A}_k, \tag{8}$$

e. g. see Konopleva — Popov book [7]. The difference is in that we use real time variable.

The units are chosen so as velocity of light in vacuum $c = 1$, hence $\Box = \partial_0^2 - \partial_k \partial_k$.

Quantization is performed in Faddeev — Popov works [4] and presented in details icluding perturbation technique in Faddeev L. [13]. Recently we evaluated correction to the mass for the Nahm reduction of YM theory by means of quasiclassical asymptotics [10; 12] developing its renormalization in [16] with applications to the special case of Heisenberg chain equation, that differs from Nahm case only by physical origin and rescaling.

3. Regularization (renormalization) as expalnation of nonzero mass appearance by quantization Faddeev: «Sidney Coleman coined a nice name *dimensional transmutation* for the phenomenon, which I am going to describe. Let us see what all this means».

«Through these (free particles) solutions are introduced via well defined quantization of the free fields. However the more thorough approach leads to the corrections, which take into account the selfnteraction of particles» [13].

4. The task of the present work is the derivation and solution of the field equations for a class of the two dimensional models. The result of the reduction of the basic YM equations and the corresponding Lagrangian is similar to the one-dimensional one [10]: we obtain 1 + 1 ϕ^4 (phi-in-quadro) model equations with the zero mass term and coefficients that depend on algebraic closure of a matrix anzatz for the gauge fields that fix the model. The stationary and directed waves (Sec.(3)) are thought as quasiperiodic solutions of the model equations that are expressed in terms of elliptic functions. Its quantization (Sec.(5)) is again performed by means reduced Lagrangean (Sec.(4)) for quasiclassical Feynman — Maslov integral, which evaluation and quantum corrections to action (Sec.(6.3)) is based on the mentioned technique of the generalized zeta-function renormalization in terms of the nonlinear Drach equation (Sec. (6.1)). It is derived for the Green function diagonal (within the heat kernel formalism) and gives polynomial solutions in elliptic variables.

Extra variables of arbitrary dimensions (Sec.(5.4), App.) are accounted for the model applications of the solutions in elementary particles physics.

2. The case of 1 + 1 dimension and reductions

2.1. *General equations in the vector form and Nahm reduction*

In 1 + 1 space, the classical YM theory [11], yields Eq. (3) with the Hamilton reduction $T_0 = 0$, that gives

$$\begin{gathered}\Box T_k + \partial_k \partial_s T_s - [T_s, \partial_k T_s] + 2[T_s, \partial_s T_k] + [\partial_s T_s, T_k] - \\ -[T_s, [T_s, T_k]] = 0,\end{gathered} \tag{9}$$

Nahm reduction $T_s = A\alpha_s$ simplifies it as

$$\begin{gathered}\alpha_k \Box A + \alpha_s \partial_k \partial_s A - [A\alpha_s, \partial_k A\alpha_s] + 3A\partial_s A[\alpha_s, \alpha_k] - \\ -A^3[\alpha_s, [\alpha_s, \alpha_k]] = 0,\end{gathered} \tag{10}$$

that fails in 1+1. Namely, taking k = 1

$$\begin{gathered}\alpha_1 \Box A + \alpha_1 \partial_1 \partial_1 A - [A\alpha_1, \partial_1 A\alpha_1] + 3A\partial_1 A[\alpha_1, \alpha_1] - \\ -A^3[\alpha_s, [\alpha_s, \alpha_1]] = \alpha_1 \partial_0^2 A - [\alpha_s, [\alpha_s, \alpha_1]]A^3 = 0,\end{gathered} \tag{11}$$

one arrive at ODE, while for k=2 we have

$$\begin{gathered}\alpha_2 \Box A + \alpha_1 \partial_2 \partial_1 A - [A\alpha_1, \partial_2 A\alpha_1] + 3A\partial_1 A[\alpha_1, \alpha_2] - \\ -A^3[\alpha_s, [\alpha_s, \alpha_2]] = \alpha_2 \Box A + +3A\partial_1 A[\alpha_1, \alpha_2] - A^3[\alpha_s, [\alpha_s, \alpha_2]] = 0,\end{gathered} \tag{12}$$

that necessarily reduces to 1D case.

2.2. *Novel reduction*

We use the Lorentz gauge, more natural for waves description and for the vector form (7) as more transparent. So, let us consider alternative (compared to Nahm one) proposal of reduction: the field is specially prepared as

$$\vec{A}_k = \phi_k(x,t)\vec{s}_k, \tag{13}$$

where $\vec{s}_k$ are constant vectors in isotopic space. It may mean that a particle space state component is linked with the isotopic one. Plugging (13) in (7) and returning to low indices, write

$$\begin{gathered}\Box\phi_k\vec{s}_k+2\varepsilon\vec{A}_0\times(2\partial_0\phi_k\vec{s}_k-\partial_k\vec{A}_0-2\varepsilon\phi_k\vec{s}_k\times\vec{A}_0)-\\-2\varepsilon\phi_j\vec{s}_j\times(2\partial_j\phi_k\vec{s}_k-\partial_k\phi_j\vec{s}_j-2\varepsilon\phi_k\vec{s}_k\times\phi_j\vec{s}_j)=0.\end{gathered}\tag{14}$$

The Eq. (8) in 1+1 reads

$$\partial_0\vec{A}_0=\partial_1\phi_1\vec{s}_1,$$

so, taking the Eq. (7) along the reduction, we write

$$\begin{gathered}\Box\phi_k\vec{s}_k+2\varepsilon\partial_0^{-1}\partial_1\phi_1\vec{s}_1\times(2\partial_0\phi_k\vec{s}_k-\\-\partial_k\vec{\partial}_0^{-1}\partial_1\phi_1\vec{s}_1-2\varepsilon\phi_k\vec{s}_k\times\partial_0^{-1}\partial_1\phi_1\vec{s}_1)-\\-2\varepsilon\phi_j\vec{s}_j\times(2\partial_j\phi_k\vec{s}_k-\partial_k\phi_j\vec{s}_j-2\varepsilon\phi_k\vec{s}_k\times\phi_j\vec{s}_j)=0.\end{gathered}\tag{15}$$

Scalar product of (15) with $\vec{s}_k$ gives

$$\begin{gathered}\Box\phi_k(\vec{s}_k,\vec{s}_k)+4\varepsilon^2\phi_k(\partial_0^{-1}\partial_1\phi_1)^2(\vec{s}_k,\vec{s}_1\times(\vec{s}_k\times\vec{s}_1))+\\+4\varepsilon^2\phi_k\phi_j\phi_j(\vec{s}_k,\vec{s}_j\times(\vec{s}_k\times\vec{s}_j))=0,\end{gathered}\tag{16}$$

because $(\vec{s}_k,\vec{s}_k\times s_1)=0$. Or, finally

$$\Box\phi_k+4\varepsilon^2C_{0k}\phi_k(\partial_0^{-1}\partial_1\phi_1)^2+4\varepsilon^2C_{1kj}\phi_k\phi_j\phi_j=0,\tag{17}$$

where

$$C_{0k}=(\vec{s}_k,\vec{s}_1\times(\vec{s}_k\times\vec{s}_1))/(\vec{s}_k,\vec{s}_k),C_{1kj}=(\vec{s}_k,\vec{s}_j\times(\vec{s}_k\times\vec{s}_j))/(\vec{s}_k,\vec{s}_k)$$

or, for normalized $\vec{s}_k$,

$$C_{0k}=1-(\vec{s}_k,\vec{s}_1)^2,C_{1kj}=(\vec{s}_k,\vec{s}_k-\vec{s}_j(\vec{s}_j,\vec{s}_k))=1-(\vec{s}_j,\vec{s}_k)^2.$$

Note also that

$$C_{02}=1-(\vec{s}_2,\vec{s}_1)^2=C_{121},C_{03}=1-(\vec{s}_3,\vec{s}_1)^2=C_{131}.$$

Plugging it in (18) gives

$$\begin{gathered}\Box\phi_1+4\varepsilon^2(1-(\vec{s}_j,\vec{s}_1)^2)\phi_1\phi_j\phi_j=0,\\\Box\phi_2+4\varepsilon^2C_{02}\phi_2(\partial_0^{-1}\partial_1\phi_1)^2+4\varepsilon^2C_{12j}\phi_2\phi_j\phi_j=0,\\\Box\phi_3+4\varepsilon^2C_{03}\phi_3(\partial_0^{-1}\partial_1\phi_1)^2+4\varepsilon^2C_{13j}\phi_k\phi_j\phi_j=0,\end{gathered}\tag{18}$$

where

$$C_{02}=1-(\vec{s}_2,\vec{s}_1)^2=C_{121},C_{03}=1-(\vec{s}_3,\vec{s}_1)^2=C_{131}.$$

For orthonormal vectors $(s_i, s_k) = \delta_{ik}$, one have

$$C_{0k} = 1 - \delta_{1k}, C_{1kj} = 1 - \delta_{jk}, \tag{19}$$

that yields

$$\Box\phi_k + 4\varepsilon^2(1-\delta_{1k})\phi_k(\partial_0^{-1}\partial_1\phi_1)^2 + 4\varepsilon^2(1-\delta_{jk})\phi_k\phi_j\phi_j = 0, \tag{20}$$

or, expanding

$$\Box\phi_k + 4\varepsilon^2(1-\delta_{1k})\phi_k(\partial_0^{-1}\partial_1\phi_1)^2 + 4\varepsilon^2\phi_k\phi_j\phi_j - 4\varepsilon^2\phi_k^3 = 0. \tag{21}$$

The system reads

$$\begin{aligned} &\Box\phi_1 + 4\varepsilon^2\phi_1(\phi_2^2 + \phi_3^2) = 0, \\ &\Box\phi_2 + 4\varepsilon^2\phi_2(\partial_0^{-1}\partial_1\phi_1)^2 + 4\varepsilon^2\phi_2(\phi_1^2 + \phi_3^2) = 0, \\ &\Box\phi_3 + 4\varepsilon^2\phi_3(\partial_0^{-1}\partial_1\phi_1)^2 + 4\varepsilon^2\phi_3(\phi_1^2 + \phi_2^2) = 0. \end{aligned} \tag{22}$$

A choice of $\phi_1 = 0$, gives

$$\begin{aligned} &\Box\phi_2 + 4\varepsilon^2\phi_2\phi_3^2 = 0, \\ &\Box\phi_3 + 4\varepsilon^2\phi_3\phi_2^2 = 0. \end{aligned} \tag{23}$$

The minimal choice in (13) is

$$\phi_1 = 0, \phi_2 = \phi_3 = \phi.$$

It is the superposition in spin and isospin states. Then, for the $k = 1$ we obtain zero identity, for $k = 2, 3$ we have the same equations of known ϕ^4 model with zero mass.

$$\Box\phi + 4\varepsilon^2\phi^3 = 0. \tag{24}$$

It is the case that is maximally close to the Nahm one, but in 1 + 1.

3. Towards a solution

3.1. Projecting technique application

Consider an equation

$$\Box\phi = F(\phi), \tag{25}$$

for arbitrary dependence in the r. h. s. Denoting

$$\phi_t = u, \phi_x = v \tag{26}$$

gives the system

$$\begin{aligned} u_t - v_x &= F\left(\int_{-\infty}^{x} v(y)dy\right), \\ v_t - u_x &= 0. \end{aligned} \tag{27}$$

The projectors [14]

$$P_{\pm} = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix} \qquad (28)$$

split the linearized system (27) in d'Alembert manner. The identity

$$(P_1 + P_2)\psi = \psi \qquad (29)$$

reads as transformation of fields and its inverse.

$$\Pi = \frac{1}{2}(u+v),$$
$$\Lambda = \frac{1}{2}(u-v). \qquad (30)$$

Acting by the projectors on the evolution system (27) yields

$$\Pi_t - \Pi_x = \frac{1}{2}F\left(\int_{-\infty}^{x}(\Pi - \Lambda)dy\right),$$
$$\Lambda_t + \Lambda_x = \frac{1}{2}F\left(\int_{-\infty}^{x}(\Pi - \Lambda)dy\right), \qquad (31)$$

that describes interaction of essentially one-dimensional waves — gives a *next step to the Nahm model*. Asymptotically, for a localized in space solutions, otherwise for a specified initial data $\Lambda = 0$ we have

$$\Pi_t - \Pi_x = \Pi_\xi = \frac{1}{2}F\left(\int_{-\infty}^{x}(\Pi)dy\right),$$
$$\pi_{\xi\xi} = \frac{1}{2}F(\pi)), \qquad (32)$$

if $\Pi = \pi_\xi, \xi = \frac{1}{2}(x-t), \eta = \frac{1}{2}(x+t)$.

In the case of the Eq. (35) it looks as nonlinear cubic oscillator

$$\pi_{\xi\xi} = 4\varepsilon^2\pi^3. \qquad (33)$$

The equation (33) has elliptic solutions [6], see details in the Sec. (6.1).

3.2. A path to wavetrains as eventually particles wavefunctions

Just remind that the anzatz with $\beta \ll 1, \beta x = x', \beta t = t'$,

$$\phi = A(\beta x, \beta t)\exp i[kx - \omega t] + c.c., \qquad (34)$$

after plugging in (24) and holding nonlinear resonance terms (e.g. [10]) in the first order by the small parameter β yields, taking into account the dispersion relation $\omega = \pm k$ and with the rule

$$A_t = \beta A_{t'}. \qquad (35)$$

It leads to the integrable (in fact - ordinary) equation

$$A_t - A_x = \frac{6\varepsilon^2}{ik} A^* A^2, \tag{36}$$

that could be solved in terms of elliptic functions.

In the case of (23) one can obtain approximate solution by the similar anzatz with $\beta << 1$,

$$\phi = A(\beta x, \beta t)\exp i[kx - \omega t] + c.c., \phi_3 = B(\beta x, \beta t)\exp i[kx - \omega t]. \tag{37}$$

The same manipulations in the first order by the small parameter β yields

$$\begin{aligned} A_t - A_x &= \frac{6\varepsilon^2}{ik}(A^* B^2 + B^* A^2), \\ B_t - B_x &= \frac{6\varepsilon^2}{ik}(B^* A^2 + A^* B^2). \end{aligned} \tag{38}$$

It is also solvable as a system of ODE.

4. Lagrange density reductions

The Lagrangian density is equal to (see, e. g. [2])

$$\mathcal{L} = \frac{1}{4}\vec{T}_{\mu\nu}\vec{T}^{\mu\nu} = \frac{1}{2}(\vec{T}_{0i}\vec{T}^{0i} + \frac{1}{2}\vec{T}_{ik}\vec{T}^{ik}) = \frac{1}{2}(\frac{1}{2}\vec{T}_{ik}\vec{T}_{ik} - \vec{T}_{0i}\vec{T}_{0i}), \tag{39}$$

the fields are normalized as in [7]. The definition (2) of the tensor $\vec{T}_{\mu\nu}$ components with account for Lorentz gauge (8)

$$\vec{A}_0 = \partial_0^{-1}\partial_k \vec{A}_k \tag{40}$$

gives

$$\vec{T}_{\mu\nu} = \vec{A}_{\nu,\mu} - \vec{A}_{\mu,\nu} - 2\varepsilon[\vec{A}_\mu \times \vec{A}_\nu], \tag{41}$$

see again [7]. The time-space components of the tensor are

$$\vec{T}_{0i} = \partial_0^{-1}\partial_k \frac{\partial \vec{A}_k}{\partial x_i} - \frac{\partial \vec{A}_i}{\partial t} - 2\varepsilon[(\partial_0^{-1}\partial_k \vec{A}_k) \times \vec{A}_i], \tag{42}$$

the sum by k is implied. The 3D subtensor looks as (41). The reduction (13) reads

$$\vec{T}_{0i} = \partial_0^{-1}\partial_k \frac{\partial \phi_k}{\partial x_i}\vec{s}_k - \frac{\partial \phi_i}{\partial t}\vec{s}_i - 2\varepsilon(\partial_0^{-1}\partial_k \phi_k)\phi_i[\vec{s}_k \times \vec{s}_i] \tag{43}$$

and

$$T_{ik} = \frac{\partial \phi_i(x)}{\partial x_k}\vec{s}_i - \frac{\partial \phi_k(x)}{\partial x_i}\vec{s}_k - 2\varepsilon\phi_i(x)\phi_k[\vec{s}_i \times \vec{s}_k]. \tag{44}$$

Its 1 + 1 space version for the SU(2) gauge (compare with [12]) gives

$$\vec{T}_{0i} = \partial_0^{-1}\frac{\partial^2\phi_1}{\partial x^2}\vec{s}_1\delta_{i1} - \frac{\partial\phi_i}{\partial t}\vec{s}_i - 2\varepsilon(\partial_0^{-1}\frac{\partial\phi_1}{\partial x})\phi_i[\vec{s}_1\times\vec{s}_i] \tag{45}$$

and

$$\vec{T}_{ik} = \frac{\partial\phi_i}{\partial x}\delta_{k1}\vec{s}_i - \frac{\partial\phi_k}{\partial x}\delta_{i1}\vec{s}_k - 2\varepsilon\phi_i\phi_k[\vec{s}_i\times\vec{s}_k]. \tag{46}$$

Then

$$\begin{aligned}\vec{T}_{0i}\vec{T}_{0i} &= (\partial_0^{-1}\frac{\partial^2\phi_1}{\partial x^2}\vec{s}_1\delta_{i1} - \frac{\partial\phi_i}{\partial t}\vec{s}_i - 2\varepsilon(\partial_0^{-1}\frac{\partial\phi_1}{\partial x})\phi_i[\vec{s}_1\times\vec{s}_i])\\ &(\partial_0^{-1}\frac{\partial^2\phi_1}{\partial x^2}\vec{s}_1\delta_{i1} - \frac{\partial\phi_i}{\partial t}\vec{s}_i - 2\varepsilon(\partial_0^{-1}\frac{\partial\phi_1}{\partial x})\phi_i[\vec{s}_1\times\vec{s}_i]).\end{aligned} \tag{47}$$

In the case of $\phi_1 = 0$,

$$\vec{T}_{0i}\vec{T}_{0i} = \frac{\partial\phi_i}{\partial t}\frac{\partial\phi_i}{\partial t}(\vec{s}_i\cdot\vec{s}_i). \tag{48}$$

Similarily

$$\begin{aligned}\vec{T}_{ik}\vec{T}_{ik} &= \frac{\partial\phi_i}{\partial x}\vec{s}_i(\frac{\partial\phi_i}{\partial x}\vec{s}_i - \frac{\partial\phi_1}{\partial x}\delta_{i1}\vec{s}_1 - 2\varepsilon\phi_i\phi_1[\vec{s}_i\times\vec{s}_1]) -\\ &-\frac{\partial\phi_k}{\partial x}\vec{s}_k(\frac{\partial\phi_1}{\partial x}\delta_{k1}\vec{s}_1 - \frac{\partial\phi_k}{\partial x}\vec{s}_k - 2\varepsilon\phi_1\phi_k[\vec{s}_1\times\vec{s}_k]) -\\ &-(2\varepsilon\phi_i\phi_k[\vec{s}_i\times\vec{s}_k])(\frac{\partial\phi_i}{\partial x}\delta_{k1}\vec{s}_i - \frac{\partial\phi_k}{\partial x}\delta_{i1}\vec{s}_k - 2\varepsilon\phi_i\phi_k[\vec{s}_i\times\vec{s}_k])\end{aligned} \tag{49}$$

and, for normalized $\vec{s}_i$,

$$\begin{aligned}\vec{T}_{ik}\vec{T}_{ik} &= \frac{\partial\phi_i}{\partial x}\frac{\partial\phi_i}{\partial x} - \frac{\partial\phi_1}{\partial x}\frac{\partial\phi_1}{\partial x} -\\ &-\frac{\partial\phi_1}{\partial x}\frac{\partial\phi_1}{\partial x} + \frac{\partial\phi_k}{\partial x}\frac{\partial\phi_k}{\partial x}\\ &4\varepsilon^2\phi_i^2\phi_k^2[\vec{s}_i\times\vec{s}_k][\vec{s}_i\times\vec{s}_k].\end{aligned} \tag{50}$$

Evaluating, $[\vec{s}_i\times\vec{s}_k][\vec{s}_i\times\vec{s}_k] = 1-(\vec{s}_i\cdot\vec{s}_k)^2 = 1-\delta_{ik}$, one arrives at

$$\vec{T}_{ik}\vec{T}_{ik} = 2\sum_2^3\frac{\partial\phi_i}{\partial x}\frac{\partial\phi_i}{\partial x} + 4\varepsilon^2(\phi_i^2\phi_k^2 - \phi_k^2\phi_k^2). \tag{51}$$

For the case of $\phi_1 = 0$, one have

$$\vec{T}_{ik}\vec{T}_{ik} = 2(\frac{\partial\phi_2}{\partial x}^2 + \frac{\partial\phi_3}{\partial x}^2) + 8\varepsilon^2\phi_2^2\phi_3^2. \tag{52}$$

Finally, the Lagrange function is

$$\mathcal{L} = -\frac{1}{2}\frac{\partial\phi_i}{\partial t}\frac{\partial\phi_i}{\partial t} + \frac{1}{2}[(\frac{\partial\phi_2}{\partial x})^2 + (\frac{\partial\phi_3}{\partial x})^2] + 2\varepsilon^2\phi_2^2\phi_3^2. \tag{53}$$

In the case $\phi_2 = \phi_3 = \phi$ it is simplified as

$$\mathcal{L} = -\phi_t^2 + \phi_x^2 + 2\varepsilon^2\phi^4(x). \tag{54}$$

It is coinside with one of classical ϕ^4 model case, derived and used, after reduction in [12] for quasiclassical correction theory. The Euler equation for (54) coincides with (24).

5. Generalized zeta-function regularization of Maslov continual integral

5.1. Action integral expansion

The energy evaluation is based on calculation of the evolution operator determinant. Its divergence is compensate by a special choice of the theory basic parameters using a freedom in the definitions. We briefly explain its origin as well as the small parameter, proportional to $\sim \hbar$ used in the quasi-classical expansion.

The approach was presented by Maslov in [17].The action functional on a quantum vector field $\vec{\phi} = \{\phi_\alpha, \alpha = 2,3\} \in \mathbb{H}$ is defined as integral over space-time stripe $t \in [0,\tau], \vec{x} \in \mathbb{R}^d$

$$S(\vec{\phi}) = \int_0^\tau\int_{\mathbb{R}^d}\left(\frac{1}{2}\left(\frac{\partial\vec{\phi}}{\partial t}\right)^2 - \sum_{n=1}^{d}\frac{1}{2}\left(\frac{\partial\vec{\phi}}{\partial x_n}\right)^2 - V(\vec{\phi})\right)d\vec{x}dt. \tag{55}$$

We adjust the regularization (renormalization) scheme [10; 16] to the problem under consideration, having in mind the Lagrange function (53). The regularization consists of two steps. First is based on the assumption, that for a vacuum state the corrections should vanish [17].

Let us expand the action integral around a specific classical field $\vec{\varphi}$ over 1 + d space-time.

$$\vec{\phi} = \vec{\varphi} + \sum_j w_j\vec{\chi}_j \tag{56}$$

with the appropriate basis $\vec{\chi}_j$ and approximate (55) as

$$S(\vec{\phi}) = S(\vec{\varphi}) + \sum_{j,k}\frac{\partial^2 S}{\partial w_j \partial w_k}(\vec{\varphi})w_j w_k + \ldots, \tag{57}$$

that, in turn, defines quasiclassical form of the path integral

$$\int_{\mathbb{H}} e^{\frac{i}{\hbar}S(\vec{\phi})} D\vec{\phi} \approx e^{\frac{i}{\hbar}S(\vec{\varphi})} \int_{\mathbb{R}} e^{\frac{i}{\hbar}\sum_{j,k}\frac{\partial^2 S}{\partial w_j \partial w_k}(\vec{\varphi}) w_j w_k} \prod_f dw_f \tag{58}$$

with $\vec{\varphi}$ as the classical path with boundary conditions $\vec{\varphi}(0,\vec{x})$, $\vec{\varphi}(\tau,\vec{x})$ and $\vec{\chi}_j$ as a basis.

Plugging (56) into (55), we obtain for the second derivative

$$\frac{\partial^2 S}{\partial w_j \partial w_k}(\vec{\varphi}) = \int_0^\tau \int_{\mathbb{R}^d} \left(\frac{\partial \vec{\chi}_j}{\partial t} \frac{\partial \vec{\chi}_k}{\partial t} - \sum_{n=1}^{d} \frac{\partial \vec{\chi}_j}{\partial x_n} \frac{\partial \vec{\chi}_k}{\partial x_n} - V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi}) \vec{\chi}_j \vec{\chi}_k \right) d\vec{x} dt, \tag{59}$$

where

$$V_{\vec{\varphi}\vec{\varphi}} \vec{\chi}_j \vec{\chi}_k = V_{\varphi_\alpha \varphi_\beta} \chi_{j\alpha} \chi_{k\beta}. \tag{60}$$

For the basic functions $\vec{\chi}_k$ from a Hilbert space $\mathbb{H}$

$$\frac{\partial^2 S}{\partial w_j \partial w_k}(\vec{\varphi}) = \int_0^\tau \int_{\mathbb{R}^d} \left(\sum_{n=1}^{d} \frac{\partial^2 \vec{\chi}_j}{\partial x_n^2} - \frac{\partial^2 \vec{\chi}_j}{\partial t^2} - V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi}) \vec{\chi}_j \right) \vec{\chi}_k d\vec{x} dt. \tag{61}$$

5.2. Rescaling the integral

Let us denote τ, λ as time and space scale parameters, ε is used as interaction parameter. The equations of motion as (24) determine a link between them. Introducing dimensionless variables $\vec{x} = \lambda \vec{x'}$, $t = \tau t'$ we rescale as

$$\frac{\partial^2 S}{\partial w_j \partial w_k}(\vec{\varphi}) = \frac{\lambda^d}{\tau} \int_0^1 \int_{\mathbb{R}^d} \left(\sum_{n=1}^{d} \frac{\tau^2}{\lambda^2} \frac{\partial^2 \vec{\chi}_j}{\partial x_n'^2} - \frac{\partial^2 \vec{\chi}_j}{\partial t'^2} - \tau^2 V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi}) \vec{\chi}_j \right) \vec{\chi}_k d\vec{x'} dt'. \tag{62}$$

The factor by the integral defines the quasiclassical expansion parameter, its value being small, allows to cut the expansion at some level. A link between λ and τ is found either from evolution equation (38) (dispersion relation in classical mechanics) or from realation between momentum, energy and mass in quantum theory. To be sure that a contribution of the last term is also of order one, we use a link between scale in time τ and constant of interaction ε that is defined in rather ambiguous way via renormalization procedure (see Sec. 6.3).

Jumping back into (58) we write the internal factor as

$$\int_{\mathbb{H}} e^{\frac{i\lambda^d}{\tau\hbar} \sum_{j,k} \int_0^1 \int_{\mathbb{R}^d} \left(\frac{\tau^2}{\lambda^2} \sum_{n=1}^{d} \frac{\partial^2 \vec{\chi}_j}{\partial x_n'^2} - \frac{\partial^2 \vec{\chi}_j}{\partial t'^2} - \tau^2 V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi}) \vec{\chi}_j \right) \vec{\chi}_k d\vec{x'} dt' w_j w_k} \prod_f dw_f, \tag{63}$$

where $V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi})$ acts as prescribed by (60)

$$[V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi})\vec{\chi}_j]_\alpha = V_{\varphi_\alpha\varphi_\beta}\chi_{j\beta}. \tag{64}$$

In the case of the Lagrangean (53) the matrix V in the isotopic subspace

$$\begin{aligned} V_{11} = V_{\varphi_2\varphi_2} &= 4\varepsilon^2\phi_3^2, \\ V_{12} = V_{21} = V_{\varphi_2\varphi_3} &= 4\varepsilon^2\phi_2\phi_3, \\ V_{22} = V_{\varphi_3\varphi_3} &= 4\varepsilon^2\phi_2^2. \end{aligned} \tag{65}$$

Transformations in both spaces ϕ and χ are changing definition of a principal state of the theory. So, if one substitute

$$\chi_{j\beta} = a^j_{\alpha\beta}\eta^j_\beta, \tag{66}$$

so that

$$V^j_{\alpha\beta}\eta^j_\beta = v(\vec{\phi})\eta^j_\alpha. \tag{67}$$

The determinant of the matrix V is zero, hence eigenvalues are

$$v_0 = 0, v_1 = 4\varepsilon^2(\phi_2^2 + \phi_3^2). \tag{68}$$

The self-action of the new basic states

$$\eta_1^{J0} = -\frac{\phi_3}{\phi_2}\eta_2^{j0}, \eta_1^{j1} = -\frac{\phi_2}{\phi_3}\eta_2^{j1} \tag{69}$$

is defined by correspondent equations that yields in different mass corrections for the principle fields.

5.3. *The final action: spectral zeta function*

The second step of the renormalization is following; introduce a new normalization parameter r of the basic functions in the Maslov integral construction [17]. We can rewrite the integral by introducing a scalar product

$$(\vec{\eta}_k, \vec{\eta}_j) = \int_0^1\int_{\mathbb{R}}\vec{\eta}_k^*\vec{\eta}_j d\vec{x'}dt' = r^{-2}\delta_{jk} \tag{70}$$

and an operator

$$\mathcal{L} = -\frac{i\lambda^d}{\pi\hbar r^2\tau}\left(\sum_{n=1}^{d}\frac{\tau^2}{\lambda^2}\frac{\partial^2}{\partial x_n'^2} - \frac{\partial^2}{\partial t'^2} - \tau^2 V_{\vec{\varphi}\vec{\varphi}}(\vec{\varphi})\right). \tag{71}$$

The quasiclassical (Maslov) functional integral (58) is written as

$$e^{\frac{i}{\hbar}S(\vec{\varphi})}\int_{\mathbb{H}} e^{-r^2\pi\sum_{j,k}(\vec{\eta}^k,\mathcal{L}\vec{\eta}^j)w_j w_k}\prod_f dw_f. \tag{72}$$

For the Hermitian $\mathcal{L}$ the eigen basis chosen yields

$$e^{\frac{i}{\hbar}S(\varphi)}\int_{\mathbb{R}} e^{-\pi\sum_j \lambda_j w_j^2} \prod_f dw_f \tag{73}$$

and, after Gauss integrals evaluation,

$$e^{\frac{i}{\hbar}S(\varphi)}\prod_j (\lambda_j)^{-\frac{1}{2}}, \tag{74}$$

having in mind that the zero values do not contribute, and the degeneracy of the eigenvalues λ_j account, formally,

$$\frac{e^{\frac{i}{\hbar}S(\varphi)}}{\sqrt{\det[\mathcal{L}]}}. \tag{75}$$

To rewrite the determinants of both operators in a form, which allow the subtraction, we use a generalized zeta-function:

$$\zeta_{\mathcal{L}}(s) = \sum_j \lambda_j^{-s}, \tag{76}$$

where λ_j are nonzero eigenvalues of $\mathcal{L}$. Such definition of the generalized zeta-function should be interpreted as analytic continuation to the complex plane of s from the half plane $\exists\sigma, \Re s > \sigma$ in which the sum converges. The right side derivative relation with respect to s at the point $s = 0$ define the determinant

$$\ln(\det\mathcal{L}) = \zeta'_{\mathcal{L}}(0). \tag{77}$$

The generalized zeta-function (76) admits the representation via the Green function of the operator $\partial_\tau + \mathcal{L}$. A link to the Green function diagonal elements (*heat kernel formalism*) has been used in quantum theory since works by Fock (1937) [18]. The zeta function in 1+d space is constructed through a set of transformations on the heat equation Green function, τ, t, t' extended for the whole axis

$$\left(\frac{\partial}{\partial\tau} + \mathcal{L}\right) g_{\mathcal{L}}\left(\tau, t, t', \vec{x}, \vec{x}'\right) = \delta(\tau)\delta(t - t')\delta\left(\vec{x}, \vec{x}'\right)). \tag{78}$$

Boundary conditions on the Green function are chosen the same as for the base functions in Maslov representation and an additional condition is applied

$$\forall_{\tau<0}\ g_{\mathcal{L}}\left(\tau, t, t', \vec{x}, \vec{x}'\right) \equiv 0. \tag{79}$$

The freedom in a vacuum choice allows to divide $\mathcal{L}=L+L_0$. In the case of 1 + 1 space, it yields for (68)

$$L=-\frac{i\lambda}{2\pi\hbar r^2\tau}\left(\frac{\tau^2}{\lambda^2}\frac{\partial^2}{\partial x'^2}-\frac{\partial^2}{\partial t'^2}+4\tau^2\varepsilon^2(\phi_2^2+\phi_3^2)-C\right), \quad (80)$$

while the function

$$L_0=-\frac{i\lambda}{2\pi\hbar r^2\tau}\left(\frac{\tau^2}{\lambda^2}\frac{\partial^2}{\partial x'^2}-\frac{\partial^2}{\partial t'^2}-C\right) \quad (81)$$

defines the vacuum part, that should be extracted as the first step of a renormalization. Here the constant C depends on the particular classical solutions that form the potential $4\tau^2\varepsilon^2(\phi_2^2+\phi_3^2)$ minimum value. We can build the renormalized zeta function by the extraction as the first step:

$$\zeta(s)=\frac{1}{\Gamma(s)}\int_0^\infty\tau^{s-1}\int_0^1\int\Big(g_L(\tau,t,t,x,x)-g_{L_0}(\tau,t,t,x,x)\Big)dxdtd\tau, \quad (82)$$

while the second step of the renormalization is realized by the special choice of the normalization constant r.

5.4. Extra variables

Working with a 1+d space, the calculations are organized as follows. For construction of the generalized zeta function it is convenient to use the property (see appendix for details)

$$g_{L_a+L_b}=g_{L_a}g_{L_b}, \quad (83)$$

valid for the operators $L_{a,b}$ dependent on different variables.

It is also useful to introduce an additional function

$$\gamma_{L_a}(\tau)=\int_0^1\int g_{L_a}(\tau,t',t',\vec{x'},\vec{x'})d\vec{x'}dt', \quad (84)$$

for which (83) holds as well. For **one-dimensional classical problem solutions**

$$L_1=A\left(\frac{\partial^2}{\partial x_1'^2}-4\lambda^2\varepsilon^2(\phi_2^2+\phi_3^2)\right), \quad (85)$$

where

$$A=-\frac{i\tau}{2\pi\hbar r^2\lambda} \quad (86)$$

with the explicit form of the classical problem solution $\varphi_i(x')$ already specified. For the first renormalization it is enough to restrict L_0 to the space variable only.

$$L_2 = -Ac^2 \frac{\partial^2}{\partial t'^2}, L_3 = A\sum_{n=2}^{3} \frac{\partial^2}{\partial x_n'^2}, \tag{87}$$

where $c^2 = \frac{\lambda^2}{\tau^2}$.

Integrating in (84) we derive the expressions for γ_{L_a}

$$\gamma_{L_2} = \sqrt{\frac{\tau}{4\pi A\lambda^2}}, \tag{88}$$

$$\gamma_{L_3} = -\frac{\lambda}{4\pi A\tau}. \tag{89}$$

We transform the Green function g_{L_1} for (118) to extract A out of it rescaling $\tau = \frac{\tau_A}{|A|}$, and arriving at

$$\left(\frac{\partial}{\partial \tau_A} - 4i\lambda^2\varepsilon^2(\phi_2^2 + \phi_3^2)\right) g_{L_1}(\tau_A, x, x_0) = \\ = \delta(\tau_A)\delta(x - x_0). \tag{90}$$

Final form of zeta function is

$$\zeta(s) = |A|^{-s} \frac{ic\lambda^{\frac{d-1}{2}}\pi^{\frac{d}{2}}}{2^d\Gamma(s)} \\ \int_0^\infty \tau_A^{s-\frac{d+2}{2}} \int \left(g_{L_1}(\tau_A, x, x) - g_{L_0}(\tau_A, x, x)\right) dx d\tau_A. \tag{91}$$

At this point **the renormalizing factor** r^2 (often referred to as the mass scale) is chosen to cut out all logarithmically divergent terms arising from differentiation of A^{-s} and possibly the Mellin integral as well. It is important to stress, that the choice of its value has impact on quantitative results and is not necessarily obvious. On purely mathematical level r^2 can be viewed as a free parameter of the theory (a reason for keeping it unspecified in works of Konoplich?), but it seems so only because we usually are unable to construct the whole propagator, which would allow us to use the normalizing condition of the propagation operator. The propagator

$$\forall_{0<T'<T} < \psi_T | e^{-\frac{i}{\hbar}TH} | \psi_0 >= \\ = \sum_{\psi_{T'}} < \psi_T | e^{-\frac{i}{\hbar}(T-T')H} | \psi_{T'} >< \psi_{T'} | e^{-\frac{i}{\hbar}T'H} | \psi_0 >, \tag{92}$$

properly set the value of r^2 (checked by a sample use of the method in the case of harmonic oscillator). Yet, to obtain physically relevant results one has to find a way of estimating the normalising factor. This problem will be discussed in phi-4 context.

6. Elliptic solutions of Nahm-like reduced model

6.1. Drach equation

The asymptotic of the solutions of the (24) is found via (33). In turn, the equation (33) that is a rescaling of the Nahms' one [6], that accounts the sel-faction constant ε. Inverse rescaling $\phi = \varepsilon\pi$ gives

$$\phi''(z) = 4\phi^3. \tag{93}$$

Integrating (93) includes a constant of integration (parameter) b

$$(\phi')^2 = (\phi^2)^2 - b^4. \tag{94}$$

It corresponds to the case m = 0 of the stationary ϕ^2 model. Solution of Nahm equation — inversion of the elliptic (lemniscate) integral yields the Jacobi sn function with the imaginary module $k = i$: $\phi = b\,sn(ibz, i)$.

$$\int_0^{\phi} \frac{d\phi}{\sqrt{\phi^4 - b^4}} = \frac{1}{b}\int_0^{\frac{\phi}{b}} \frac{dt}{\sqrt{(t^2-1)(t^2+1)}} = z, \tag{95}$$

so the constant b enters the solution as amplitude and space scale parameter.

6.2. Drach equation for the Green function diagonal

Take a Laplace transform $\hat{g}_L(p, x, x_0)$ of the Green function, defined by (78): $G(p, x) = \hat{g}_L(p, x, x)$ is a solution of bilinear equation [10]

$$2GG'' - (G')^2 - 4(u(x) - p)G^2 + 1 = 0. \tag{96}$$

Such equation was introduced by J. Drach in other context [15]. In a case of reflectionless and finite-gap solutions is solved via polynomials (in p) P, Q

$$G(p, x) = P(p, z) / \sqrt{Q(p, z)}, \tag{97}$$

where $z = cn^2(bx; k)$. Plugging (97) yields

$$b^2(\rho(2PP'' - (P')^2) + \rho' PP') - (p + u))P^2 + Q = 0, \tag{98}$$

the primes denote derivatives with respect to z, while

$$\begin{aligned} \rho(z) &= z(1 - z)(2 - z), \\ u(z) &= -6b^2(1 - z). \end{aligned} \tag{99}$$

The polynomials

$$\begin{aligned} P &= p^2 + P_1(z)p + P_2(z), \\ Q &= p^5 + q_4 p^4 + q_3 p^3 + q_2 p^2 + q_1 p + q_0. \end{aligned} \tag{100}$$

A substitution of (100) into the equation splits in the system

$$\begin{gathered}-2P_1-u+q_2=0,\\ -2P_2-P_1^2-2uP_1+b^2(2\rho P_1''+\rho' P_{1'})+q_3=0,\\ b^2(\rho(2P_{2''}+2P_1P_{1''}-(P_{1'})^2)+\\ +\rho'(P_{2'}+P_1P_{1'}))-2P_1P_2)-u(2P_2+P_1^2)+q_2=0,\\ b^2(2\rho(2P_{1''}P_2+P_{1'}P_{2'}+P_1P_{2''})+\\ +\rho'(P_1P_{2'})+P_{1'}P_2))-P_2^2-2uP_1P_2+q_1=0,\\ b^2(\rho(2P_2P_{2''}-P_{2'}^2))+\rho' P_2^2+q_0=0.\end{gathered} \tag{101}$$

The arguments in (101) are omitted. The case of **Nahm equation** yields [10]

$$q_4=0,\quad q_3=-21b^4,\qquad q_2=q_1=108b^8, q_0=0,$$

hence $P_1(z)=-3b^2(z-1),\quad P_2=18b^4z^2-36b^4z.$

$$Q(p)=p(p+3b^2)(-p+3b^2)(12b^4-p^2), \tag{102}$$

where the polynomial Q simple roots p_i are ordered for real b. Finally,

$$Q=\prod_{i=1}^{i=5}(p-p_i), \tag{103}$$

where the polynomial Q have the simple roots p_i and reflection symmetry $\rightarrow reduction$.

6.3. Mass as the correction

Let us pick up the expressions determining $\hat{\gamma}(p)$, integrating by period:

$$\hat{\gamma}(p)=\int(p^2-3b^2(z-1)p+18b^4z^2-36b^4z)dx/2\sqrt{Q}. \tag{104}$$

Going to the variable z, and integrating,

$$\begin{gathered}\hat{\gamma}(p)=[6b^4K(i)+2p^2K(i)+\\ +36b^4\left(K(i)-E(i)\right)-3b^2p\left(E(i)-3K(i)\right)]/\sqrt{Q},\end{gathered} \tag{105}$$

that gives the zeta function (91) via complete elliptic lemniscate integrals K(i) and E(i).

$$\begin{gathered}\zeta(s)=\frac{icl^{\frac{d-1}{2}}\pi^{\frac{d}{2}}}{2^d\,|A|^{-s}\,\Gamma(s)}\\ \int_0^\infty \tau_A^{s-\frac{d+2}{2}}\int\left(g_{L_1}(\tau_A,x,x)-g_{L_0}(\tau_A,x,x)\right)dxd\tau_A.\end{gathered} \tag{106}$$

Or, plugging the (103) and (105),

$$\zeta(s) = -\int_l \frac{1}{(-p)^s} \frac{2K(i)p^2 + 3b^2(K(i) - E(i))p - 48b^4 K(i)}{2\sqrt{p(p+3b^2)(p-3b^2)(p-2\sqrt{3}b^2)(2\sqrt{3}b^2 + p)}} dp, \quad (107)$$

rescaling, $p = p'b^2, dp = dp'b^2$ and denote, $K(i) = K, E(i) = E,$

$$\zeta(s) = -\int_l \frac{b^{5-2s}}{(-p)^s} \frac{2Kp^2 + 3(K-E)p - 48K}{2\sqrt{p(p+3)(p-3)(p-2\sqrt{3})(2\sqrt{3}+p)}} dp, \quad (108)$$

and the final form for energy corrections

$$\Delta E = \Re\left(\lim_{s\to 0} \frac{\partial}{\partial s} \frac{\hbar c l^{\frac{d-1}{2}} \pi^{\frac{d}{2}}}{2^{d+1} T |A|^{-s} \Gamma(s)} \int_0^\infty \tau_A^{s-\frac{d+2}{2}} \int \left(g_{L_1}(\tau_A, x, x) - g_{L_0}(\tau_A, x, x) \right) dx d\tau_A \right). \quad (109)$$

Compared to the expression in [10]. Finally, the gauge field particle mass in the quasiclasical approximation is evaluated as

$$m = \zeta'(0) = \lim_{s\to 0} \frac{d}{ds} - b^{-2s} b^5 \int_l (-p)^{-s} \frac{2Kp^2 + 3(K-E)p - 48K}{2\sqrt{p(p+3)(p-3)(p-2\sqrt{3})(2\sqrt{3}+p)}} dp. \quad (110)$$

Conclusion

We have considered a nonlinear plane wave of SU(2) YM field (see e. g. [3]) in a 1 + d space. It is shown that the quantum correction to energy in quasiclassical approximation gives nonzero mass that is evaluated via hyperelliptical integral (109). Consideration of the 1 + d case in our paper allows to apply multidimension theories as of the in [20]. More generally one can apply the generalized semiclassical Foldy-Wouthuysen transformation as e. g. in [9].

Of separate interest there is the special case in the Heisenberg ferromagnet theory. It is the easy axis case when the «mass terms» tends to zero. It corresponds the special choice of the magnetic field B value.

Appendix

$$E_q = E_c + \Re\left(\frac{i\hbar}{2T} (\ln(\det[L]) - \ln(\det[L_0])) \right), \quad (111)$$

where

$$L_0 = -\frac{iTa^d}{2\pi\hbar r^2} \left(-\frac{\rho}{T^2} \frac{\partial^2}{\partial t'^2} + \frac{\varepsilon}{a^2} \sum_{n=1}^{d} \frac{\partial^2}{\partial x_n'^2} - C \right), \quad (112)$$

defines the vacuum state

$$L = -\frac{iTa^d}{2\pi\hbar r^2}\left(-\frac{\rho}{T^2}\frac{\partial^2}{\partial t'^2} + \frac{\varepsilon}{a^2}\sum_{n=1}^{d}\frac{\partial^2}{\partial x_n'^2} + V(\phi) - C\right). \tag{113}$$

To explain (83), take $\phi_{1,j}(x)$ as eigenfunctions of L_1 with eigenvalues $\lambda_{1,j}$ and $\phi_{2,n}(y)$ as eigenfunctions of L_2 with eigenvalues $\lambda_{2,n}$. Due to the independence of variables, $\phi_{1,j}(x)\phi_{2,n}(y)$ are eigenfunctions of $L_1 + L_2$ with eigenvalues $\lambda_{1,j} + \lambda_{2,n}$.

$$g_{L_1+L_2} = \sum_{j,n} e^{-(\lambda_{1,j}+\lambda_{2,n})\tau}\frac{\phi_{1,j}(x)\phi_{2,n}(y)\phi_{1,j}(x_0)\phi_{2,n}(y_0)}{(\phi_{1,j}\phi_{2,n}, \phi_{1,j}\phi_{2,n})}\Theta(\tau). \tag{114}$$

Considering the scalar product we use, we prove

$$(\phi_{1,j}\phi_{2,n}, \phi_{1,j}\phi_{2,n}) = (\phi_{1,j}, \phi_{1,j})_1(\phi_{2,n}, \phi_{2,n})_2, \tag{115}$$

$$g_{L_1+L_2} = \sum_{j} e^{-\lambda_{1,j}\tau}\frac{\phi_{1,j}(x)\phi_{1,j}(x_0)}{(\phi_{1,j}, \phi_{1,j})_1}\sum_{n} e^{-\lambda_{2,n}\tau}\frac{\phi_{2,n}(y)\phi_{2,n}(y_0)}{(\phi_{2,n}, \phi_{2,n})_2}\Theta(\tau). \tag{116}$$

It is convenient to introduce an additional function

$$\gamma_L(\tau) = \int_0^1\int g_L(\tau, t', t', \vec{x}', \vec{x}')d\vec{x}'dt', \tag{117}$$

for which (83) holds as well. For **one-dimensional classical solutions**

$$L_1 = A\left(\frac{\partial^2}{\partial x_1'^2} - \frac{a^2 V''(\varphi(x_1'))}{\varepsilon}\right), \tag{118}$$

$$A = -\frac{iT\varepsilon a^{d-2}}{2\pi\hbar r^2} \tag{119}$$

with the exact form of potential V and solution φ already specified. We restrict L_0 to the x variable only. For the rest of the presentation x_1' will be denoted as x and $\forall_n G_n = G$. Next let $c^2 = \frac{GT^2}{Ma^2}$.

$$L_2 = -\frac{A}{c^2}\frac{\partial^2}{\partial t'^2}, L_3 = A\sum_{n\neq 1}\frac{\partial^2}{\partial x_n'^2}. \tag{120}$$

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